Open problems from update meeting on graph separation problems

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Parameterized Complexity of Directed Multicut

Dániel Marx

Is Directed Multicut fixed-parameter tractable when parameterized by the number of terminals and the size of the cutset? We know that:

1. Directed Multiway Cut is FPT when parameterized by the size of the cutset only [2].
2. Directed Multicut is W[1]-hard when parameterized by the size of the cutset only [4], even in DAGs [3].
3. Directed Multicut is FPT when parameterized by the size of the cutset and the number of terminals, when the input graph is a DAG [3].
4. Directed Multicut is NP-hard and APX-hard for two terminal pairs [1], but the two-terminal case can be reduced to Directed Multiway Cut. It is open whether it is FPT for 3 terminal pairs, parameterized by the size of the cutset.

References

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Faster algorithms for finding good cuts

Marcin and Michał Pilipczuk

A (q, k)-good cut in an undirected graph G is a set X of at most k edges, such that $G \setminus X$ has exactly two connected components, each containing more than q vertices. The notion of good cuts is essential for recursive calls in the k-Way Cut algorithm by Kawarabayashi and Thorup [2] and in the follow-up technique of randomized contractions [1]. Via randomized contractions, a (q, k)-good cut can be found in roughly $\mathcal{O}^*(q^k)$ time. Can this running time be significanlty improved? A positive answer would be a first step to speed up the algorithms based on the randomized contractions technique, which currently seem to be stuck at running time $\mathcal{O}^*(2^{\mathcal{O}(k^2 \mathrm{polylog}(k))})$.

References

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Faster algorithms for Odd Cycle Transversal and related problems

Daniel Lokshtanov

The currently fastest FPT algorithm for Odd Cycle Transversal and Vertex Cover above LP runs in $\mathcal{O}^*(2.318^k)$ time [3, 2]. The base of the exponent comes from branching vectors with complicated case analysis, so we do not expect it to be optimal. Can it be significantly improved? For example, to $\mathcal{O}^*(2^k)$?

A related question is to improve the $\mathcal{O}^*(2^k)$ algorithm for Edge Bipartization [1].

References

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Polynomial kernel for Multicut in DAGs

Marcin Pilipczuk

In [1] we refuted the existence of polynomial kernels for most graph separation problems in directed graphs, as Directed Multiway Cut with 2 terminals is OR-compositional. The remaining case is the Multicut problem in directed acyclic graphs (shown to be FPT in [3]). Does it admit a polynomial kernel, when parameterized by the size of the cutset and the number of terminal pairs? Or when parameterized by the size of the cutset, with constant number of terminal pairs?

References

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Parameterized complexity of König Edge Deletion

Saket Saurabh

An undirected graph is a *König graph* if it admits a vertex cover of size equal to the size of its maximum matching. This class contains all bipartite graphs, but not every König graph is bipartite; for example, a triangle with a pentant vertex attached to one vertex is a König graph (see e.g. [2]).

Consider the König Edge Deletion problem where we are to delete as few edges as possible from the given graph to obtain a König graph. Is this problem FPT, parameterized by the number of edge deletions? Note that this problem is at least as hard as Almost 2-SAT, and the vertex deletion variant is shown to be FPT in [1].

References

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A single-exponential algorithm for Directed FVS

Marcin Pilipczuk

Since 2008 we know that Directed Feedback Vertex Set is fixed-parameter tractable, but the only known algorithm runs in $\mathcal{O}^*(k!4^k)$ time [1]. The $k!$ factor comes out from considering all orderings of the modulator set in the iterative compression step; the rest of the algorithm runs in $\mathcal{O}^*(2^{\mathcal{O}(k)})$ time. Can this step be avoided, so that DFVS would be solved in $\mathcal{O}^*(2^{\mathcal{O}(k)})$ time? Or maybe it is impossible, assuming ETH?

References

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Parameterized complexity of Interval Edge Deletion

Yixin Cao

Since late 2012 we know that Interval Vertex Deletion is fixed-parameter tractable [1, 2]. What is the parameterized complexity of the edge-deletion variant of this problem?

References

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Parameterized complexity of Stable Multicut

Michał Pilipczuk

Since 2011 we know that MULTICUT, parameterized by the size of the cutset, is fixed-parameter tractable [1, 4]. What is the parameterized complexity of the variant of this problem, when we require the cutset to be independent? Note that, when parameterized by the size of the cutset and the number of terminal pairs, the problem is FPT due to the treewidth reduction technique [2, 3].

References

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