Open problems from Workshop on Kernels

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Treedepth-d modulator as a parameter

Somnath Sikdar

The recent kernelization algorithms in very general sparse graph classes such as graphs of bounded expansion use the structural parameter of a constant-treedepth modulator [1]. The parameter seems natural in these graph classes, but we have not yet investigated its full power also in general graphs. What natural problems admit a polynomial kernel with respect to this parameter in general graphs? Are there many examples of natural problem that, under this parameterization, admit a polynomial kernel in sparse graph classes, but not in general graphs?

References

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A better kernel for treewidth-t modulator

Daniel Lokshtanov

The TREEWIDTH-t MODULATOR problem, where we are to delete at most k vertices as possible to obtain a graph of treewidth at most t, has a kernel of size $\mathcal{O}(k^{f(t)})$ [1]. Can it be improved to a kernel of size $f(t)k^c$, for c independent of t? In case of a positive answer, the next step is to improve the kernel bounds for the general PLANAR $\mathcal{F}$-DELETION problem.

References

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Tight bounds for kernels for Vertex Cover

Fedor Fomin

It seems reasonable to believe that the $2k$-vertex kernel for VERTEX COVER [4] is optimal, as a $(2-\varepsilon)$-approximation algorithm for VERTEX COVER would violate the Unique Games Conjecture [3], and it is hard to imagine a $(2-\varepsilon)k$-vertex kernel that would not yield a $(2-\varepsilon')$-approximation algorithm for VERTEX

Cover. However, the aforementioned argumentation is informal. Can we prove a matching lower bound for the $2k$-vertex kernel, assuming some widely-believed complexity assumption?

A similar question can be considered in the case of planar graphs. Here, no approximation arguments restrict us, as Vertex Cover admits a PTAS in planar graphs (via the classical Baker's approach [1]). Note that the $4k$-vertex kernel for Independent Set in planar graphs yields an $(\frac{4}{3} - \varepsilon)k$-vertex lower bound for a Vertex Cover kernel in planar graphs [2]. However, still the best known upper bound is the $2k$-vertex kernel inherited from general graphs.

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Is Treewidth OR-compositional?

Bart Jansen

The question whether the input graph has treewidth at most k, parameterized by k, is clearly AND-compositional [1] and, by the recent result of Drucker [2], most likely does not admit a polynomial kernel. However, the question of an OR-(cross)-composition to Treewidth remains open. It has been recently shown that Pathwidth is OR-compositional (unpublished).

The question is closely related to the question of a maximum size of a minimal forbidden minor for graphs of treewidth at most k: it is unknown whether the sizes of such graphs are bounded polynomially in k.

References

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[2] Andrew Drucker. New limits to classical and quantum instance compression. In *FOCS*, pages 609–618. IEEE Computer Society, 2012.

Polynomial kernel for Directed FVS

long-standing

Let us repeat the long-standing open problem of an existence of a polynomial kernel for Directed Feedback Vertex Set, parameterized by the size of the deletion set. The FPT algorithm is known since 2008 [1].

References

[1] Jianer Chen, Yang Liu, Songjian Lu, Barry O'Sullivan, and Igor Razgon. A fixed-parameter algorithm for the directed feedback vertex set problem. *J. ACM*, 55(5), 2008.

Polynomial kernel for Multiway Cut

Yixin Cao

The recent applications of matroid techniques to kernelization resulted in a $\mathcal{O}(k^{t+1})$-vertex kernel for MULTIWAY CUT with t terminals and k being the bound on the size of the cutset [1]. Can the dependency on t be removed from the exponent? The problem remains open even in the (easier) edge-deletion variant of MULTIWAY CUT.

References

[1] Stefan Kratsch and Magnus Wahlström. Representative sets and irrelevant vertices: New tools for kernelization. In *FOCS*, pages 450–459. IEEE Computer Society, 2012.

Polynomial kernel for Multicut in DAGs

Marcin Pilipczuk

(Note that this problem appears also on the open problems list from the update meeting on graph separation problems.)

In [1] we refuted the existence of polynomial kernels for most graph separation problems in directed graphs, as DIRECTED MULTIWAY CUT with 2 terminals is OR-compositional. The remaining case is the MULTICUT problem in directed acyclic graphs (shown to be FPT in [3]). Does it admit a polynomial kernel, when parameterized by the size of the cutset and the number of terminal pairs? Or when parameterized by the size of the cutset, with constant number of terminal pairs?

References

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A polynomial kernel for Knapsack

Daniel Lokshtanov, Saket Saurabh

In the KNAPSACK problem, we are given n items with sizes $(s_i)_{i=1}^n$ and values $(v_i)_{i=1}^n$ and capacity of the knapsack B; we are to choose a set of items $A \subseteq \{1, 2, \ldots, n\}$ that fit into the knapsack ($\sum_{i \in A} s_i \leq B$) and have maximum possible total value (maximize $\sum_{i \in A} v_i$). Does this problem admit a polynomial kernel with respect to parameter n? That is, can we reduce the sizes and the values, so that their bit-length is bounded polynomially in n?

The answer is affirmative (using randomization) for a related problem of Subset Sum [1]. Moreover, there exists a randomized Turing kernel for Knapsack parameterized by n [2]. More formally, the algorithm of [2] outputs ℓ Knapsack instances such that

1. the answer to the original instance is an OR of the output instances;
2. the algorithm is randomized with one-sided error (it may produce false positives);
3. ℓ is bounded polynomially in n and the bit-length of the input sizes and values; and
4. each output size and value have bit-length bounded polynomially in n.

References

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[2] Jesper Nederlof, Erik Jan van Leeuwen, and Ruben van der Zwaan. Reducing a target interval to a few exact queries. In Branislav Rovan, Vladimiro Sassone, and Peter Widmayer, editors, *MFCS*, volume 7464 of *Lecture Notes in Computer Science*, pages 718–727. Springer, 2012.

A linear element-kernel for d-Hitting Set

Saket Saurabh

The Vertex Cover problem admits a $2k$-vertex kernel [3], but is unlikely to admit a $\mathcal{O}(k^{2-\varepsilon})$-edge kernel [2]. More generally, we know that the d-Hitting Set admits a kernel with $\mathcal{O}(k^d)$ sets and $\mathcal{O}(k^{d-1})$ elements [1], and a matching lower bound for the number of sets is known [2]. However, it remains open whether we can further reduce the number of elements in the kernel. In particular, does d-Hitting Set admit a kernel with $f(d)k$ vertices?

References

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Line Graph Edge Deletion

Falk Hüffner

The Line Graph Edge Deletion problem asks to delete at most k edges from the input graph to obtain a line graph. The characterization by forbidden induced subgraphs yields a $\mathcal{O}^*(11^k)$-time algorithm. Can this algorithm be significantly improved? Does this problem admit a polynomial kernel?

Claw-free Edge Deletion

Michał Pilipczuk

A similar question as before can be asked for the Claw-free Edge Deletion problem. A graph is claw-free if it does not contain a $K_{1,3}$ as an induced subgraph, and every line graph is a claw-free graph. The forbidden induced subgraphs characterization immediately yields a $\mathcal{O}^*(3^k)$ FPT algorithm. What about a polynomial kernel?

Polynomial kernels for interval/chordal modification problems

Marcin Pilipczuk, Saket Saurabh

There are more graph edition problems where a question of a polynomial kernel is open.

1. Interval Vertex Deletion, shown recently to be FPT [1, 4].
2. Chordal Vertex Deletion [3].
3. Interval Completion [6].
4. Proper Interval Vertex Deletion: there is a $\mathcal{O}(k^{53})$ kernel [2] and a $\mathcal{O}^*(6^k)$ FPT algorithm [5]. Can we obtain a significantly smaller kernel with, say, at most $\mathcal{O}(k^{10})$ vertices? Magnus Wahlstöm mentioned that he obtained a progress using the matroid techniques, but is still far from the $\mathcal{O}(k^{10})$ goal.

References

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Framework for refuting Turing kernels

long-standing

Let us repeat the long-standing open problem of providing a framework for refuting Turing kernels. Currently, we know that there is a large group of problems equivalently (un)likely to have Turing kernels [1].

References

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